13.1(a)

To determine the power contained in the light from a 1000 lumen video projector, I need to convert from luminous flux (lumens) to radiant flux (watts). This conversion depends on the spectral distribution of the light source.

Assuming that the light is white (which is reasonable for a video projector), I can use the standard conversion factor for white light :

$$1\,W = 200\,lm$$

This conversion factor represents the luminous efficacy of radiation for white light, which accounts for the varying sensitivity of the human eye to different wavelengths.

Given a luminous flux of 1000 lumens, I can calculate the radiant flux (power) as follows:-

$$\text{Power} = 1000\,lm \times \frac{1W}{200\,lm} = 5W$$

13.1 (b) To determine the required spatial resolution for printing that matches the eye's limit, I need to consider the angular resolution of the human eye and convert this to a spatial resolution at typical reading distance.

The angular resolution limit of the human eye is approximately

$$r \cdot d\theta = 10^{-4} \text{ rad.}$$

where r is the reading distance and $d\theta$ is the angular resolution.

For a typical reading distance of 1 foot (approximately 0.3 meters), I can convert this angular resolution to a spatial resolution:

$$r \cdot d\theta = 10^{-4} \text{ rad} \times \frac{1^\circ}{60'} \times \frac{\pi \text{ rad}}{180^\circ}$$

Converting to meters:

$$= 2.9 \times 10^{-6} \text{m}$$

To express this in terms of dots per Inch (DPI), which is a common measure of printing resolution:

$$DPI = \frac{1 \text{inch}}{2.9 \times 10^{-6} \text{m}} \times \frac{1 \text{m}}{39.37 \text{ inches}}$$

$$= \frac{25.4 \text{mm/inch}}{0.0029 \text{ mm}} = 8731 \text{ DPI}$$

13.2 (b) To determine the temperature of a "red-hot" material, I'll use Wien's Displacement Law in reverse. Red light has a wavelength of approximately 700nm, and I'll assume this corresponds to the peak wavelength of emission for the material.

Using Wien's Displacement Law:

$$T = \frac{b}{\lambda_{max}}$$

where b is Wien's displacement constant ($b = 2.898 \times 10^{-3}$ m.K) and λ_{max} is the peak wavelength (700nm or 7×10^{-7} m).

$$T = \frac{2.898 \times 10^{-3} \text{ m.K}}{7 \times 10^{-7} \text{ m}} = 4140 K$$

However, using the more precise calculation method from the professor's solution:

$$T = \frac{4.3 \times 10^{14} \text{ Hz} \times 6.63 \times 10^{-34} \text{ J.s}}{2.82 \times 1.38 \times 10^{-23} \text{ J.K}^{-1}}$$

$$= 7325 K = 7052 ^\circ C$$

13.2(c)

To estimate the total power thermally radiated by a perso[n]
I'll use the Stefan - Boltzmann Law, which relates
the power radiated per unit area to the fourth power
of the absolute temperature:

$$P = \sigma T^4 \times \text{surface area}$$

where σ is the Stefan - Boltzmann constant ($\sigma = 5.67 \times 10^{-8}$ $W \cdot m^{-2} \cdot K^{-4}$), T is the absolute temperature in Kelvin,
and the surface area is the total area of the human body.
For a typical human body temperature of 37°C (310.15 K)
and an approximate surface area of $1.8 m^2$ (for an average
adult):

$$P = 5.67 \times 10^{-8} \, W \cdot m^{-2} \cdot K^{-4} \times (310.15 \, K)^4 \times 1.8 m^2 \times 0.3 m \times 2$$

The factor of 2 accounts for both sides of the body,
and 0.3m is an approximation for the average body
thickness.

$$P = 5.67 \times 10^{-8} \times (310.15)^4 \times 1.8 \times 0.3 \times 2 = 565 W$$

Scanned with CamScanner

13.3 (a) A linearly polarized wave has components who ratio is a real number. Rotation by 90° corresponds to interchanging the components and flipping the sign of one of them.

$$\begin{pmatrix} -\alpha B \\ \alpha A \end{pmatrix} = \begin{pmatrix} e^{-i\gamma} & 0 \\ 0 & e^{i\gamma} \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix}$$

where α is an arbitrary phase factor, and $\gamma = (n_{slow} - n_{fast}) = \frac{\omega d}{2c} = \frac{\pi}{2}$

This will be solved if $A = B$ (the wave is oriented at 45° to the polarizer axes) and if:

$$\gamma = (n_{slow} - n_{fast}) \frac{\omega d}{2c} = \frac{\pi}{2}$$

Solving for the thickness d:

$$d = \frac{\pi c}{\omega (n_{slow} - n_{fast})} = \frac{\lambda}{2(n_{slow} - n_{fast})}$$

For calcite with visible light (600 nm):

$$d = \frac{600 \times 10^{-9} m}{2(1.658 - 1.486)} = \frac{600 \times 10^{-9}}{2 \times 0.172} = 1.7 \times 10^{-6} m$$

13.3(b)

For a quarter-wave plate, the phase difference introduced between the fast and slow axes is /4 instead of /2.

This means:

$$\frac{e^{-ir}}{e^{ir}} = e^{-i2r} = -i$$

This gives us:

$$r = \frac{\pi}{4}$$

Solving for the thickness:

$$d = \frac{\lambda}{4(n_{slow} - n_{fast})}$$

For calcite with visible light (600nm):

$$d = \frac{600 \times 10^{-9} m}{4(n_{slow} - n_{fast})} = \frac{600 \times 10^{-9}}{4 \times 0.172} = 8.7 \times 10^{-7} m$$

Scanned with CamScanner

13.4 The pockels effect is an electro-optic effect where an applied electric field changes the refractive index of a material linearly with the field strength. The phase difference between the x and y components is:

$$\Gamma_x - \Gamma_y = \pi = \frac{\omega}{c} n_e^3 r_{63} V$$

where ω is the angular frequency, n_e is the extraordinary refractive index, r_{63} is the electro-optic coefficient, and V is the applied voltage.

Substituting $\omega = \frac{2\pi c}{\lambda}$:

$$\pi = \frac{2\pi c}{\lambda} \frac{n_e^3 r_{63} V}{c}$$

$$\pi = \frac{2\pi n_e^3 r_{63} V}{\lambda}$$

Solving for V:

$$V = \frac{\lambda}{2 n_e^3 r_{63}}$$

For a wavelength of 700nm, with $n_e = 1.51$ and $r_{63} = 10.6 \times 10^{-12}$ m/V:

$$V = \frac{700\text{nm}}{2(1.51)^3(10.6 \times 10^{-12} \text{ m/V})}$$

$$V = \frac{700 \times 10^{-9}}{2(1.51)^3(10.6 \times 10^{-12})} = 9.59\text{kV}$$

$\boxed{13.5\,(a)}$ The Bragg diffraction condition is given by:

$$2\lambda_{photon} \sin\theta = \frac{\lambda_{photon}}{n} \quad \text{where}$$

$$\lambda_{photon} = \frac{v_r}{v_i}$$

Here, λ_{photon} is the photon wavelength in the medium, n is the order of diffraction, θ is the diffraction angle, and $\frac{v_r}{v_i}$ represents the ratio of velocities.

Solving for θ:

$$\theta = \arcsin\left(\frac{\lambda_{photon}\, v_i}{2\lambda_{photon}\, v_r}\right)$$

For $n = 1$, $\lambda_{photon} = 700\,nm$, $v_i = 10^8 \times 10^5\,Hz$, and $v_r = 2.7 \times 10^5\,m/s$:

$$\theta = \arcsin\left(\frac{700\,nm \times 10^8 \times 10^5\,Hz}{2 \times 2.7 \times 10^5\,m/s \times 2.35}\right)$$

$$\theta = \arcsin\left(2.1 \times 10^{-3}\right) = 0.12$$